P.R.GOVT.COLLEGE (AUTONOMOUS), KAKINADA

II B.SC. – MATHEMATICS / SEMESTER- III (W.E.F. 2017-2018)

Course: ABSTRACT ALGEBRA

Total Hrs. of Teaching: 60 @ 4 h / Week **Total Credits : 03**

Objective:

- To learn about the basic structure in Algebra
- To understand the concepts and able to write the proofs to theorems
- To know about the applications of group theory in real world problems

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Module -I

Unit 1: Groups **(15 hours)**

Binary Operation – Algebraic structure – semi group – monoid –Definition and elementary properties of a Group – Finite and Infinite groups – Examples – Order of a group – Composition tables with examples.

Unit 2: Subgroups, Cosets and Lagrange's Theorem **(15 hours)**

Definition of Complex – Multiplication of two complexes – Inverse of a complex – Subgroup definition – examples - criterion for a complex to be a subgroup –criterion for the product of two subgroups to be a subgroup – union and intersection of subgroups.

Cosets definition – properties of cosets – Index of subgroup of a finite group – Lagrange's Theorem.

Module -II

Unit 3: Normal Subgroups **(10 hours)**

Definition of normal subgroup – proper and improper normal subgroup – Hamilton group – criterion for a subgroup to be a normal subgroup – intersection of two normal subgroups – subgroup of index 2 is a normal subgroup – simple group – quotient group – criteria for the existence of a quotient group.

Unit 4: Homomorphism **(10 hours)**

Definition of homomorphism – Image of homomorphism – elementary properties of homomorphism – Definition and elementary properties of Isomorphism and automorphism – Kernel of a homomorphism – Fundamental theorem on homomorphism and applications.

Unit 5: Permutations and Cyclic groups **(10 hours)**

Definition of permutation – permutation multiplication – Inverse of a permutation – Cyclic permutations – transposition – even and odd permutations – Cayley's theorem.

Definition of cyclic group - elementary properties – classification of cyclic groups.
Additional Inputs : Applications of group theory

Text Book:
Abstract Algebra by J.B.Fraleigh

Books for reference:
1 A text book of Mathematics, S.Chand and Co.
2. Modern Algebra by Gupta and Malik
3 Elements of Real Analysis by Santhi Nararayana & M.D.Raisinghania.

BLUE PRINT FOR QUESTION PAPER PATTERN
SEMESTER-III

	Unit	TOPIC	V.S.A.Q	S.A.Q	E.Q	Marks allotted to the Unit
MODULE-I	1	Groups	2	2	2	28
	2	Subgroups, Cosets & Lagrange's theorem	2	3	1	25
MODULE-II	3	Normal Subgroups	2	1	1	15
	4	Homomorphism	1	2	1	19
	5	Permutations and Cyclic groups	1	2	1	19
TOTAL			8	10	6	106

V.S.A.Q = Very short answer questions (1 mark)
S.A.Q = Short answer questions (5 marks)
E.Q = Essay questions (8 marks)

Very short answer questions : 8 X 1 =08
Short answer questions : 6 X 5 =30
Essay questions : 4 X 8 =32

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Total Marks = 70

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P.R GOVT.COLLEGE (AUTONOMOUS), KAKINADA
II B.Sc. EXAMINATION, MATHEMATICS
MODEL PAPER – III SEMESTER, PAPER-II
Core (Advanced): Abstract Algebra

Time: 3 hours Max.Marks: 70M

PART –I

Answer the following questions. Each question carries 1 mark. **8x1M =8M**

1. Write the Cauchy's composition table for $G = \{1, \omega, \omega^2\}$.
2. Find the Identity element in the group $(G,*)$ where * is defined by $a * b = \frac{ab}{3}\ \forall a, b \in G = Q/\{0\}$
3. Write a proper subgroup of a group $G = \{1, -1, i, -i\}$ with respect to multiplication.
4. How many right cosets have a sub group $H = \{0,3,6,9,12\}$ in the group $(Z_{15}, +_{15})$ where $Z_{15} = \{0,1,2, \dots, 13,14\}$.
5. Define normal subgroup.
6. Check whether $f: (Z, +) \longrightarrow (Z, +)$ defined by $f(x) = x^2$ is a homomorphism or not.
7. Write the inverse of the permutation $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 3 & 1 & 2 & 5 \end{pmatrix}$.
8. Define cyclic group.

Part-II

Answer THREE questions from each section, each question carries Five marks.

6x5M=30M

SECTION-A

9. Prove that the set Z of all integers form an abelian group w.r.t. the operation defined by $a * b = a + b + 2\ \forall\ a, b \in Z$.
10. In the group (G,.), show that the equations $ax = b, ya = b\ \forall\ a, b \in G$ have unique solutions in G.
11. Prove that a non empty complex H of a group G is a subgroup of G if and only if $H = H^{-1}$.
12. Prove that any two left (right) cosets of a subgroup H of a group G are either identical or disjoint.
13. State and Prove Lagrange's theorem.

SECTION-B

14. If M, N are two normal subgroups of G such that $M \cap N = \{e\}$ then every element of M commutes with every element of N.
15. If f is a homomorphism of a group G into a group G', then prove that the kernel of f is a normal subgroup of G.
16. A mapping f from a group $(G,.)$ to $(G,.)$ defined by $f(a) = a^{-1} \forall a \in G$ is a homomorphism if and only if G is abelian.
17. Express the product $(2 \quad 5 \quad 4)(1 \quad 4 \quad 3)(2 \quad 1)$ as a product of disjoint cycles and find its inverse.
18. Find the regular permutation group of the multiplicative group $G = \{1, \omega, \omega^2\}$.

PART-III

Answer FOUR questions from the following ; choosing at least ONE question from each section. Each question carries 8 marks. 4X8M=32M

SECTION-C

19. Show that the n^{th} roots of unity form an abelian group with respect to multiplication.
20. If 'a' is an element of a group G such that $O(a) = n$, then prove that
$$a^m = e \text{ } iff \text{ } n/m.$$
21. State and Prove the necessary and sufficient condition for a finite complex H of a group G to be a subgroup of G.

SECTION-D

22. If H is a normal subgroup of a group (G,.) then prove that the product of two right (left) cosets of H is also a right (left) coset of H.
23. Prove that every homomorphic image of a group G is isomorphic to some quotient group of G.
24. Prove that every subgroup of a cyclic group is cyclic.